Institute of Systems Engineering
Real Time Systems Group

Extrinsic Calibration Between a 3D Laser Scanner and a Camera Under Interval Uncertainty

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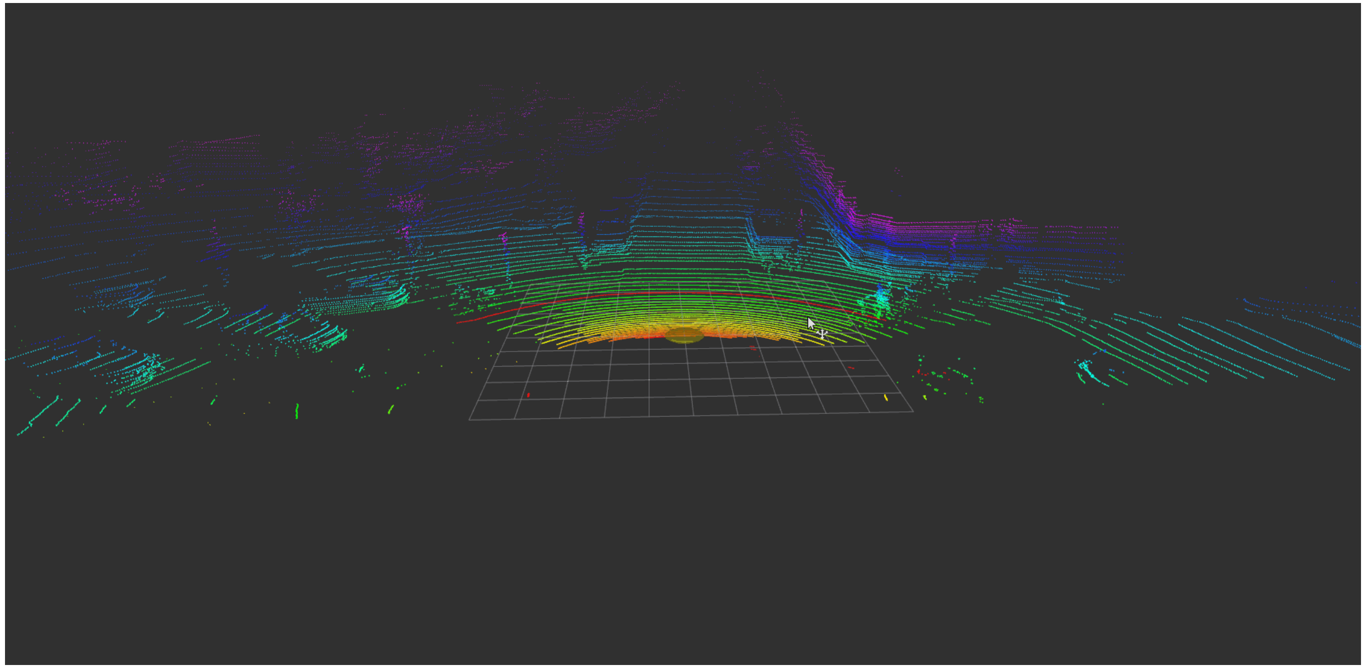
DFG Research Training Group (GRK2159)
i.c.sens - Integrity and Collaboration in dynamic sensor networks

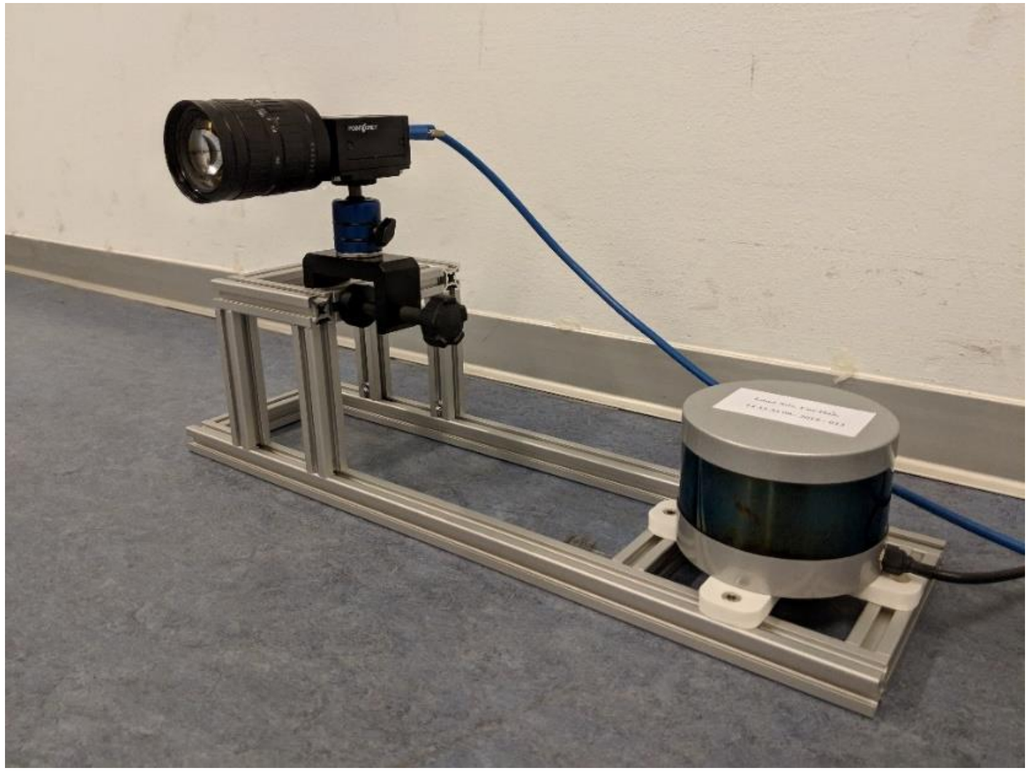
3D Camera-Laser Extrinsic Calibration

- Complementary information of the environment from camera and laser scanner
- Extrinsic parameters (rotation and translation) required to fuse information



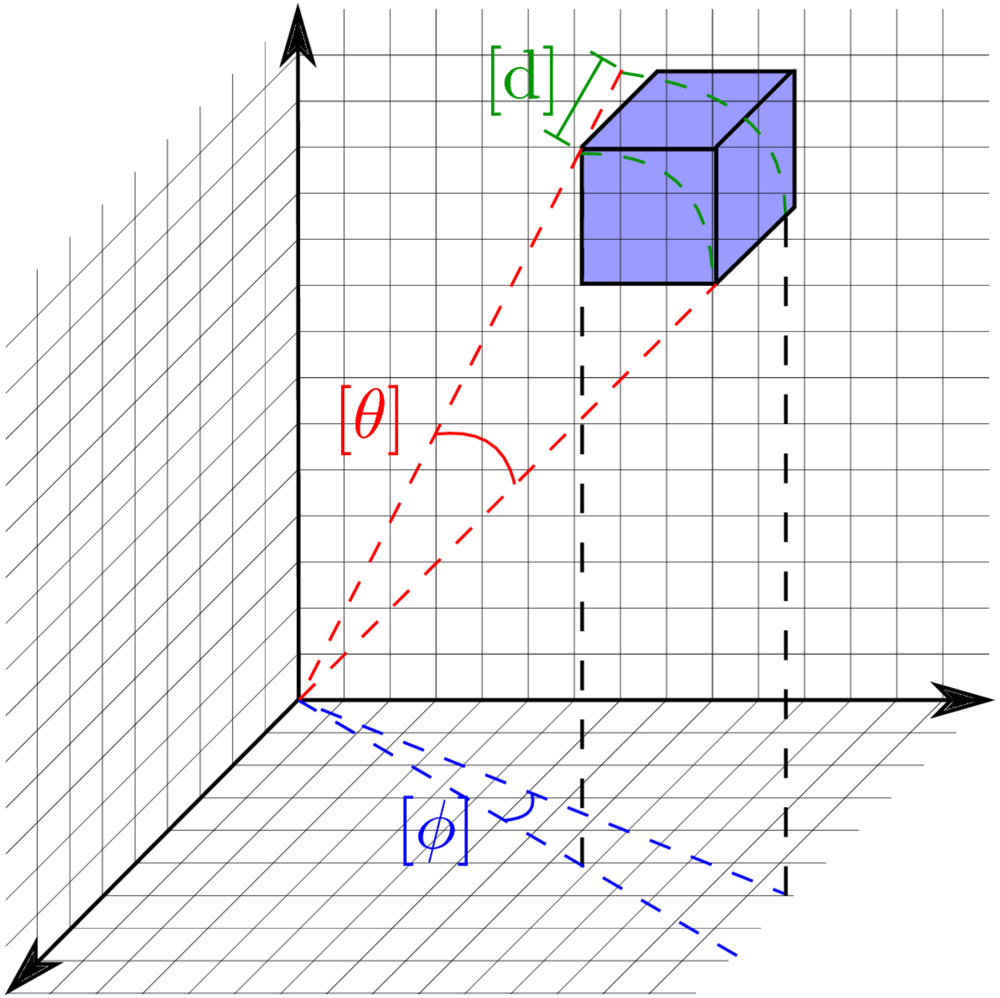
3D Camera-Laser Extrinsic Calibration

- Why intervals to model error?
  - Error distribution of sensors is often unknown (laser scanner, camera)
  - Unknown, systematic errors can be modeled (e.g. biased intrinsic camera parameters or laser distance measurements)
  - Consistent error propagation
  - Fused information is guaranteed
  - No initial value needed



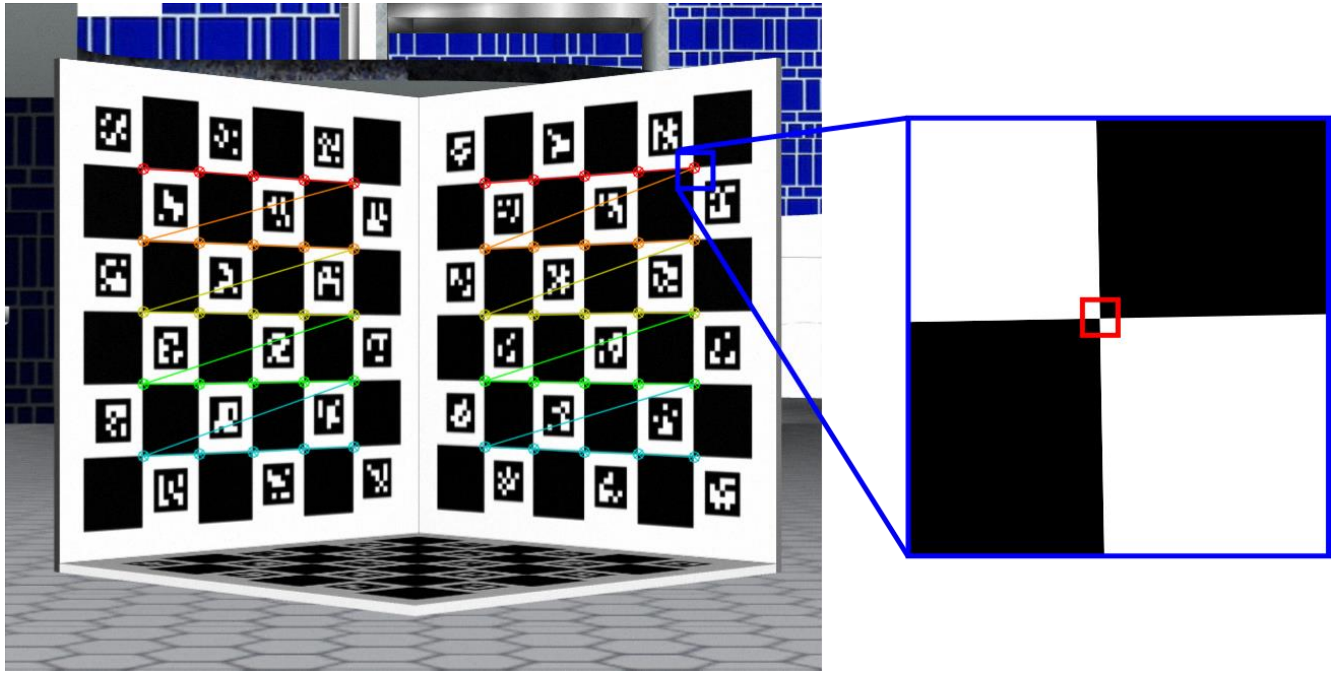
Modelling the Sensor Errors: Laser Scanner

- Interval for distance measurement: [d]
- Interval for angular components: [ϕ] and [θ]
- Computation of 3D Cartesian coordinates



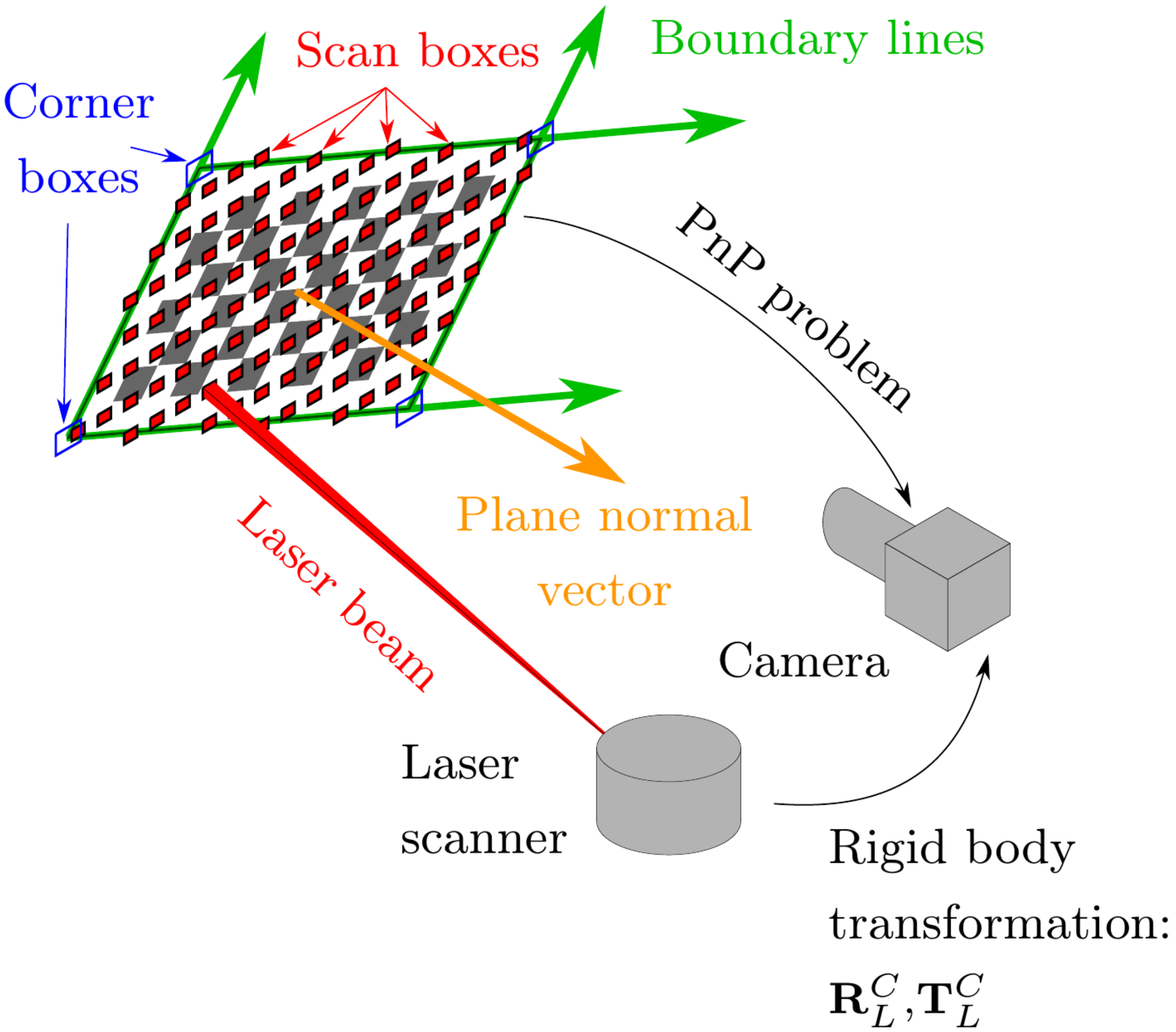
Modelling the Sensor Errors: Camera

- Interval boxes instead of point-valued feature detections
- Error bounds can be found from calibration process
  - Maximum reprojection error



General Idea

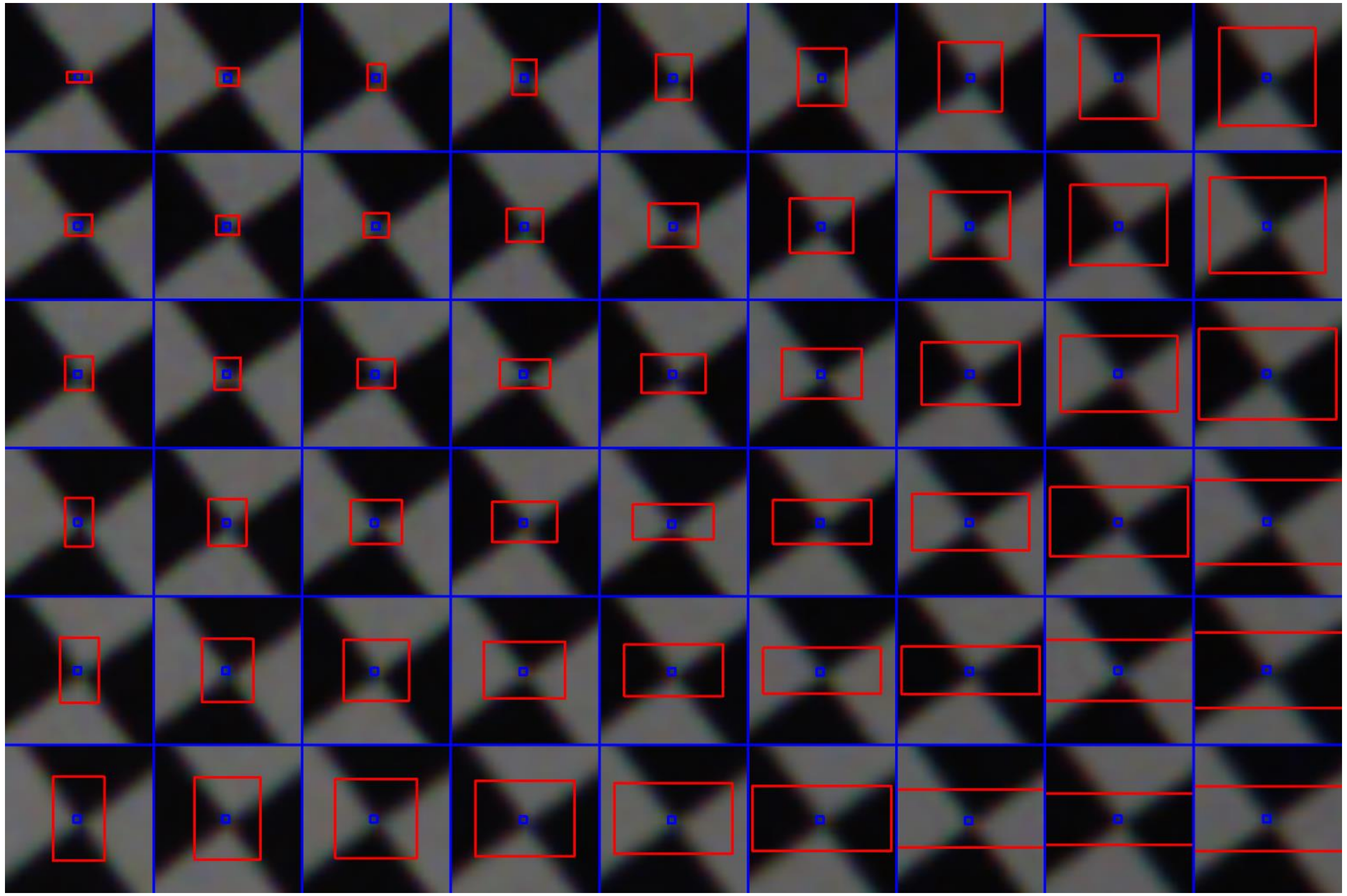
- Find corresponding features for laser scanner and camera on checkerboard
  - Plane parameters
  - Line parameters
  - 3D corner points
- Multiple checkerboard poses
- SIVIA with forward-backward contractors to find rotation and translation



Camera Feature Extraction: PnP Problem

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = R \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + T$$

- λ is the unknown scale factor
- $(u, v, 1)^T$ is a corner on the image plane
- $(x, y, z)^T$ is the corresponding 3D world point
- R (rotation) and T (translation) are the wanted extrinsic parameters
- SIVIA with contractors to find an enclosure for R and T
- No initial solution needed



Blue: detected image feature red: reprojected world point

Camera Feature Extraction: Plane Equation

- General plane equation: $ax + by + cz + r = 0$
- $(a, b, c)^T$ is the plane normal vector $(x, y, z)^{\mathrm{T}}$ is a 3D point on the plane

Known three points on the plane:

$$X_1 = R \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + T = T$$

$$X_2 = R \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + T$$

$$X_3 = R \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + T$$

Vectors on plane:

$$\overrightarrow{X_2} = X_2 - X_1 = R \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} r_{11} \\ r_{21} \\ r_{31} \end{pmatrix}$$

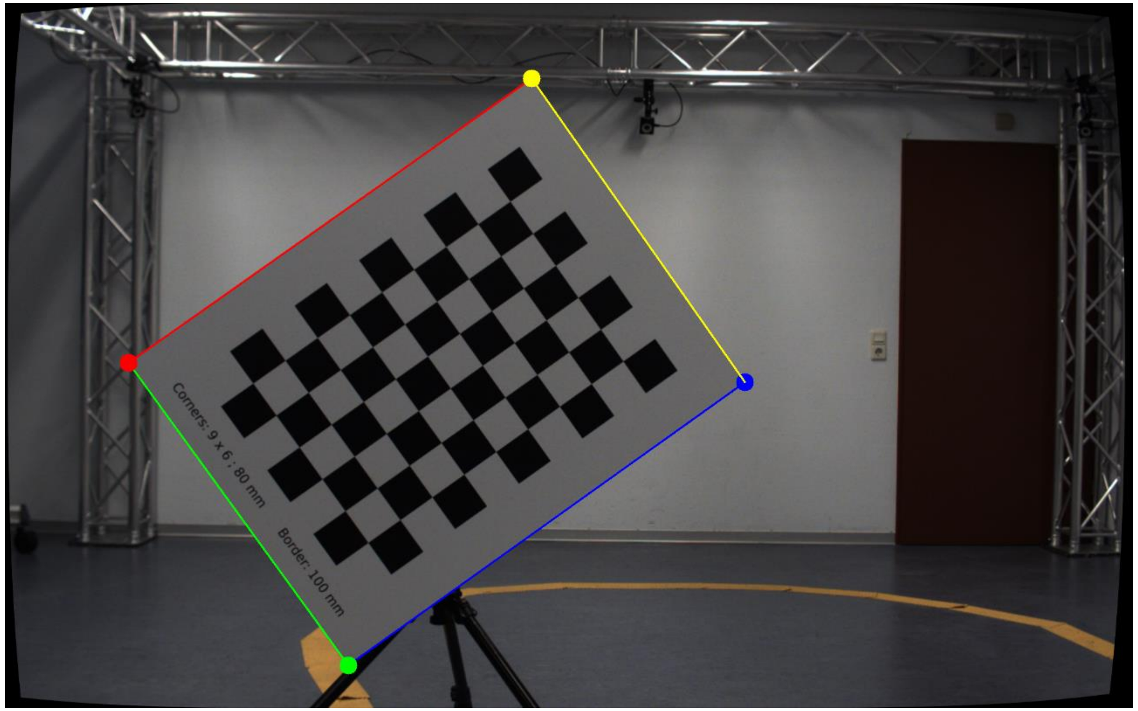
$$\overrightarrow{X_3} = X_3 - X_1 = R \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} r_{12} \\ r_{22} \\ r_{32} \end{pmatrix}$$

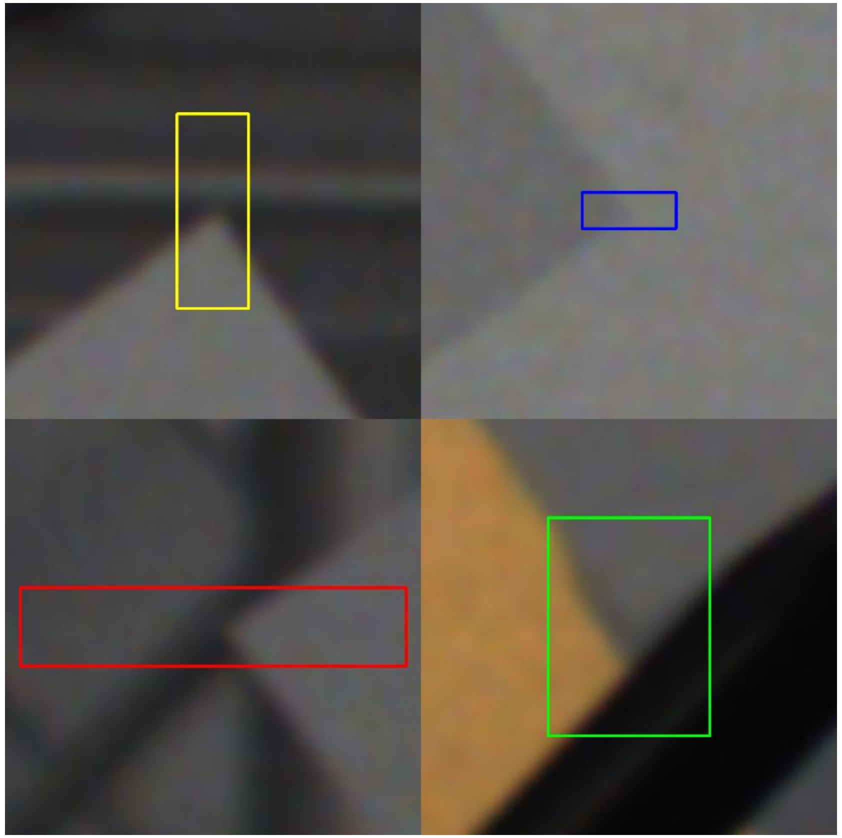
Cross product results in plane normal:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \overrightarrow{X_2} \times \overrightarrow{X_3}$$

Camera Feature Extraction: Corner Points + Line Equations

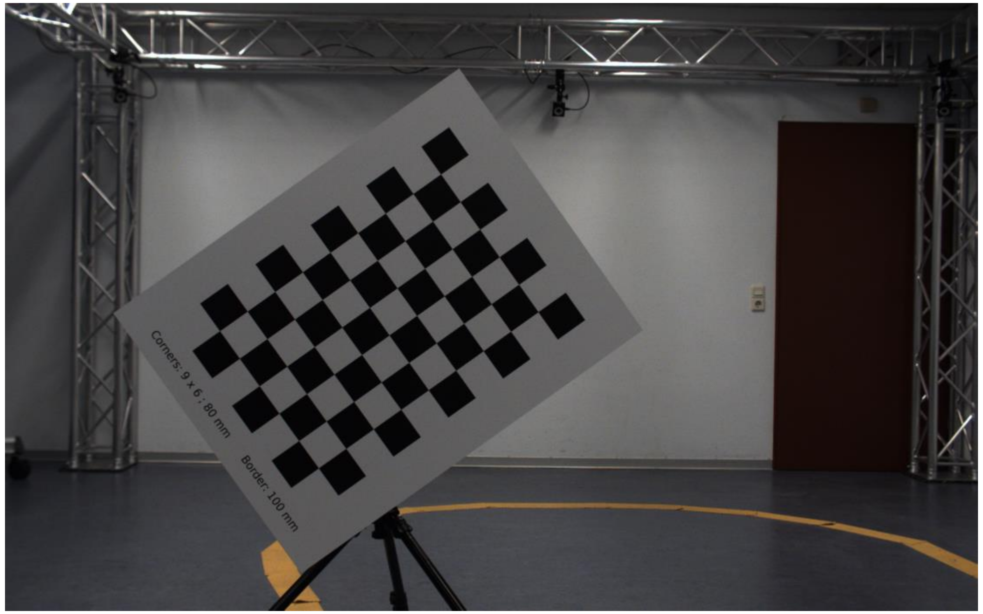
- Corner points are known in checkerboard coordinate system
- Transform corner points into camera coordinate system using R and T
- Line direction is the difference between two adjacent corner points

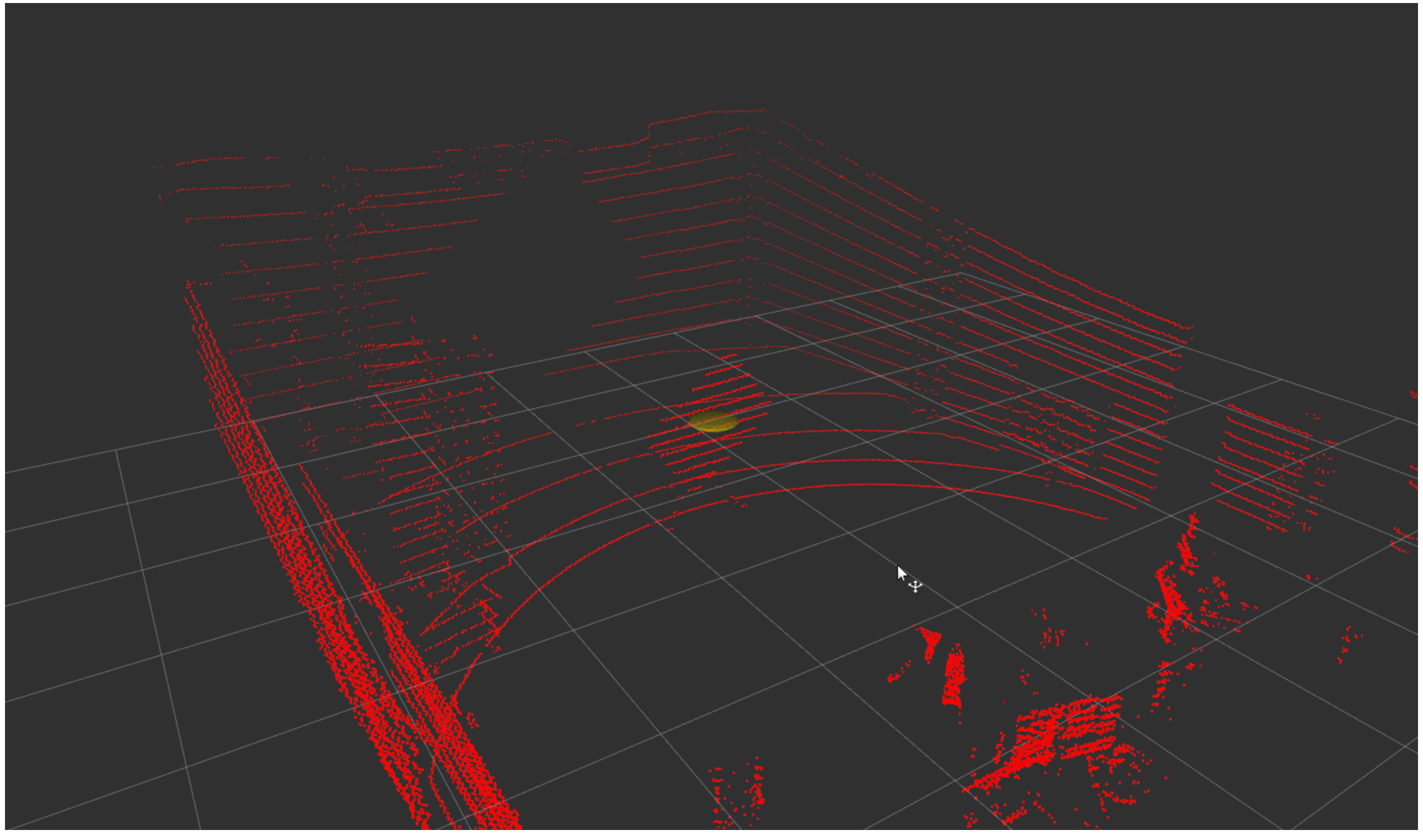




Laser Feature Extraction: Preliminary Steps

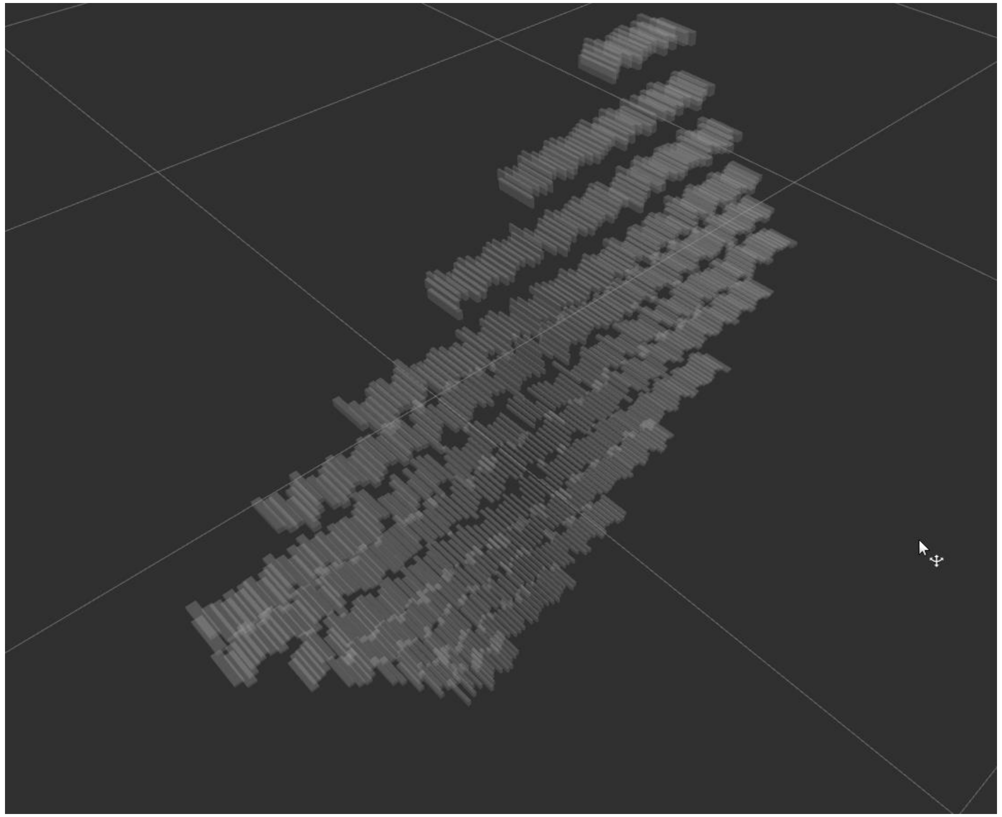
- Manually define 3D cube that contains checkerboard
- Remove scan lines with insufficient points
- Remove points on tripod using RANSAC

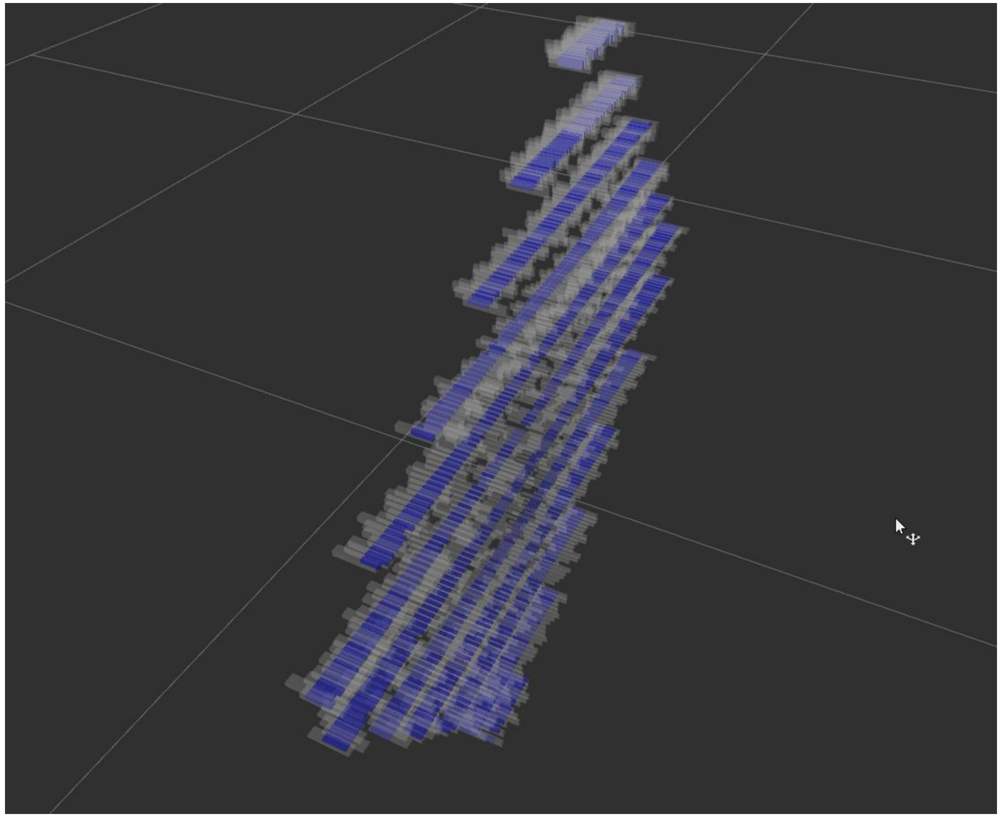




Laser Feature Extraction: Plane Fitting

- Constraints: Plane equation $ax + by + cz + r = 0$ Unit vector $a^2 + b^2 + c^2 = 1$
- SIVIA + forward-backward contractor to find domains $[a], [b], [c], [r]$



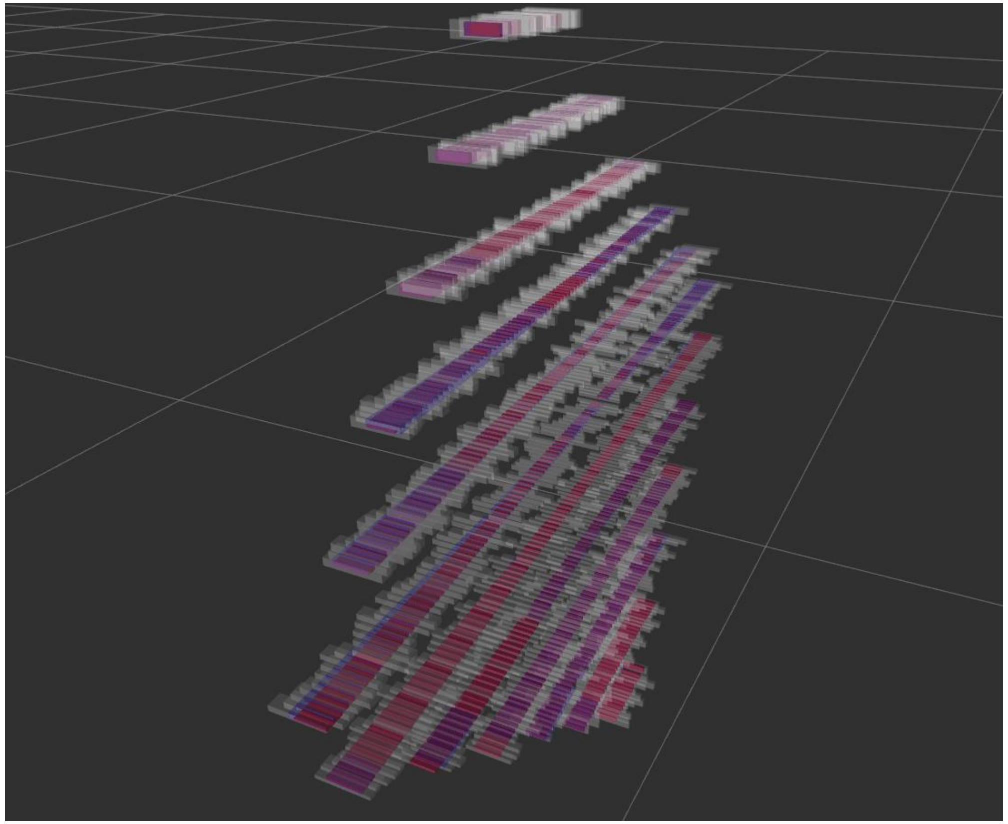


Laser Feature Extraction: Line Directions

- Wanted: scan line direction d
- Contract plane points with respect to line constraint

Points on line Scale factors

$$P_1 + r_1 \cdot d = P_2 + r_2 \cdot d$$

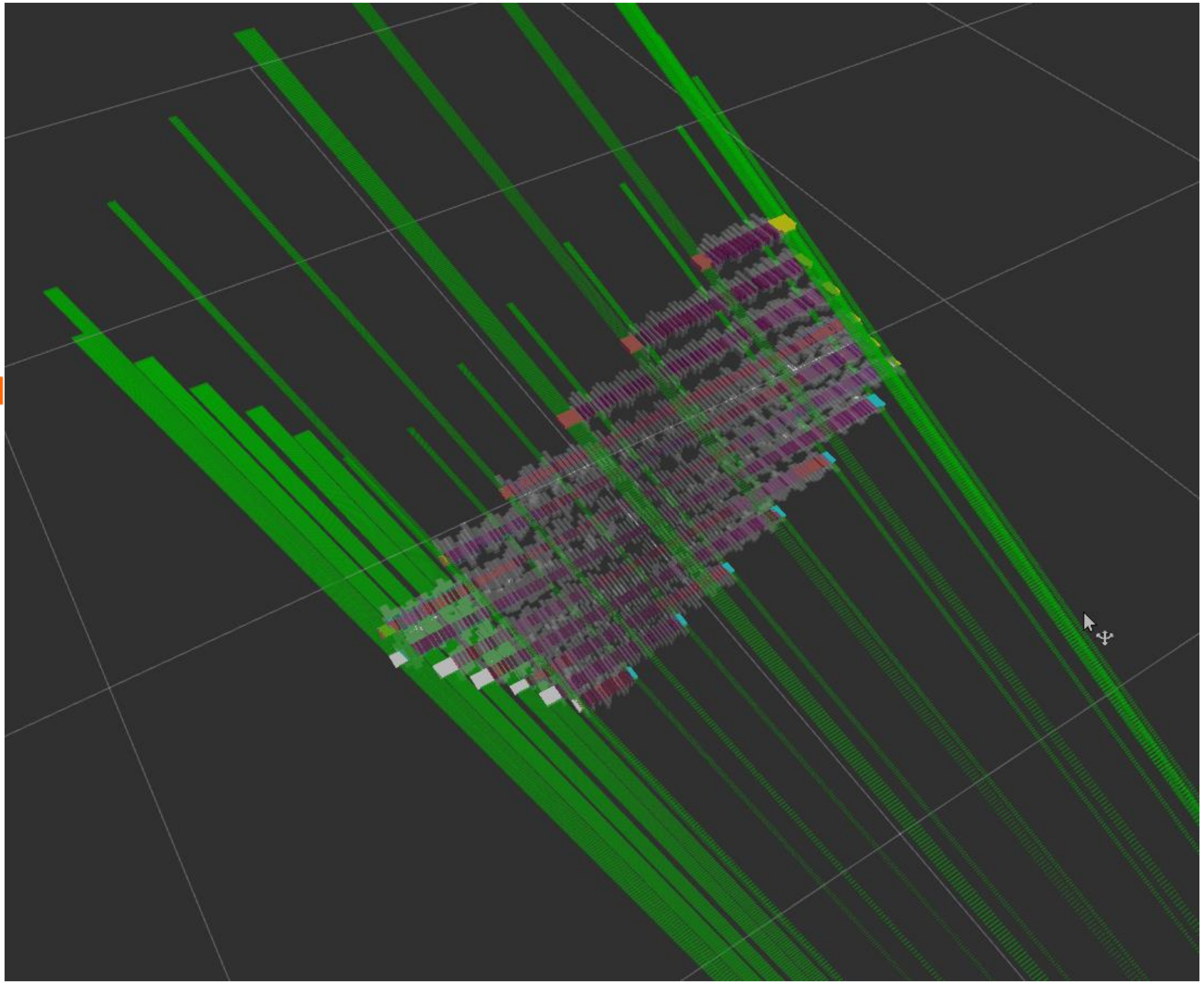


White: original boxes
blue: after plane contraction
red: after line contraction

Laser Feature Extraction: Line Directions

- Wanted: checkerboard border points
- Must be between last point on board and first point not on board
  - Intersection between two interval lines: scan line and hypothetical laser ray
  - Results in box for border point

Bord

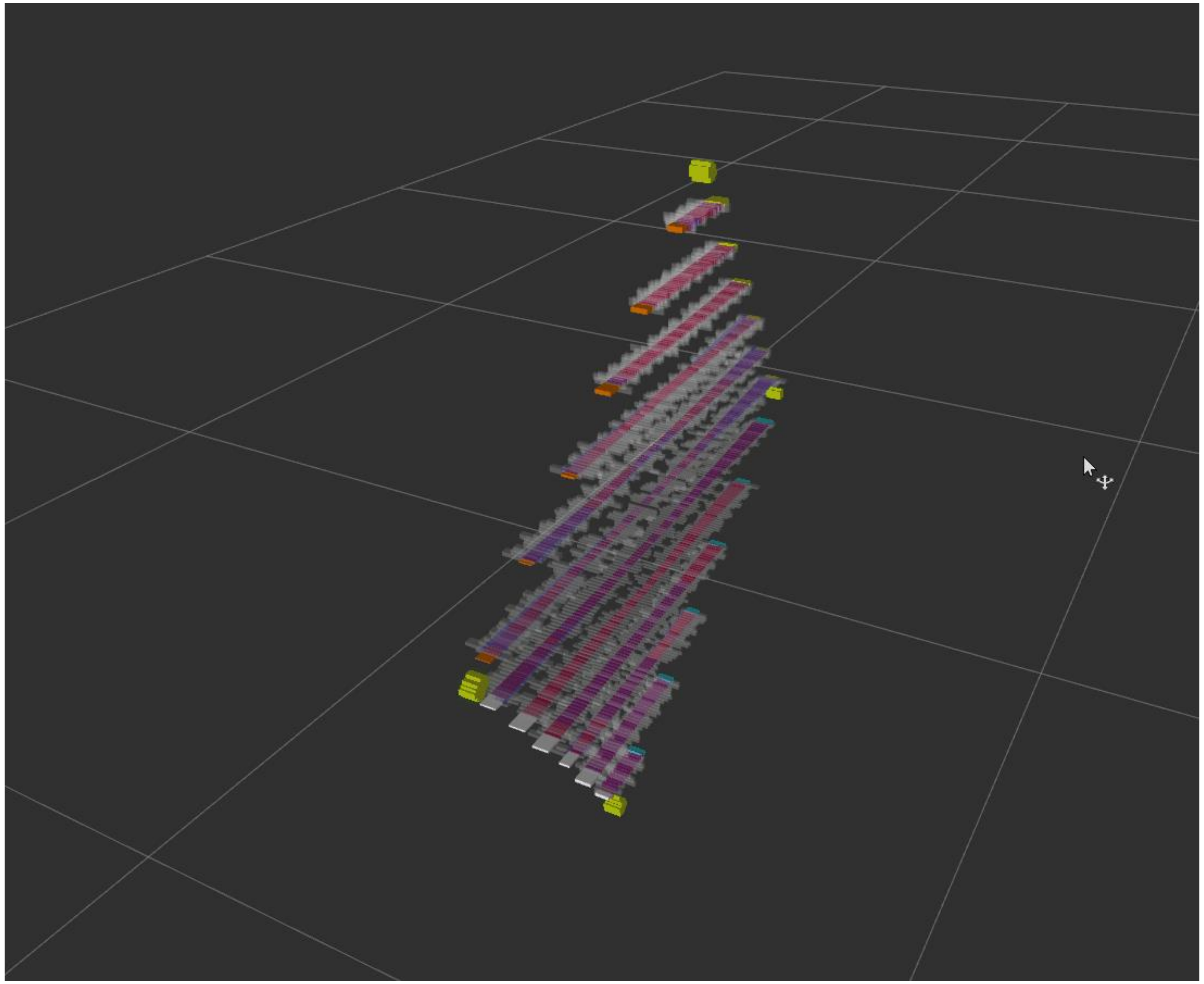


Laser Feature Extraction: Line Directions

- Wanted: checkerboard border line directions d^L and corner points C^L
- Fit line through border points to find d^L
- Interval line intersection to find C^L

Points on line   Scale factors

$$P_1 + r_1 \cdot d = P_2 + r_2 \cdot d$$



Finding the Extrinsic Parameters

- From plane correspondence:

laser scanner

camera

$$R \cdot n^L = n^C$$

rotation matrix

plane normal

translation vector

$$n^C \cdot \left(R \cdot Q_j^L + T\right) + r^C = 0$$

point on plane

plane distance

Finding the Extrinsic Parameters

- From line correspondences:

laser scanner

camera

$$R \cdot d_i^L = d_i^C$$

rotation matrix

border line direction

translation vector

$$(I_{3\times3} - d_i^C \left(d_i^C\right)^T)(R \cdot P_j^L + T - P_k^C) = 0_{3\times1}$$

identity matrix

point on line

Finding the Extrinsic Parameters

- From corner point correspondences:

rotation matrix — translation vector

$$R \cdot C_i^L + T = C_i^C$$

corner point

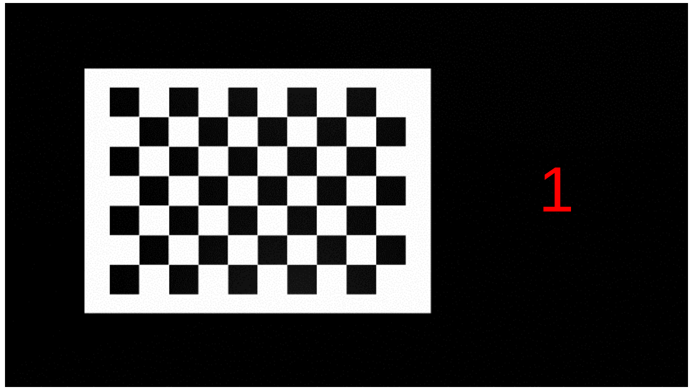
- SIVIA + forward-backward contractor to enclose domains for $[R]$ and $[T]$
- Rotation matrix $[R]$ is parametrized using Euler angles

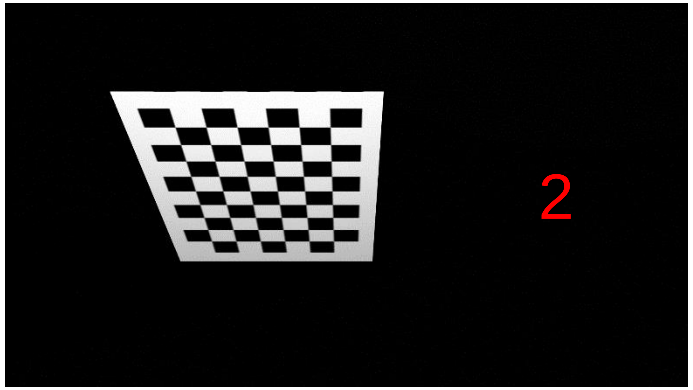
Results

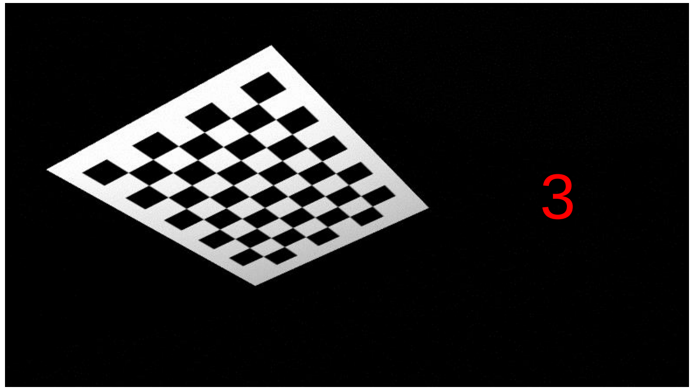
- Simulated data, three individual poses

Pose	$[\phi]$(Roll) [deg]	$[\theta]$ (Pitch) [deg]	$[\psi]$ (Yaw) [deg]	$[t_x]$ [cm]	$[t_y]$ [cm]	$[t_z]$ [cm]
True	90.0	0.0	0.0	-27.0	15.0	-12.0
1	[89.3,90.7]	[-0.6,0.5]	[-0.6,0.7]	[-29.7,-24.4]	[-35.0,65.0]	[-13.6,-10.5]
2	[89.6,90.3]	[-0.6,0.6]	[-0.7,0.7]	[-30.5,-24.1]	[-17.4,47.0]	[-62.0,38.0]
3	[89.2,90.7]	[-1.1,0.9]	[-1.1,1.2]	[-32.0,-21.2]	[11.2,18.6]	[-14.2,-9.5]

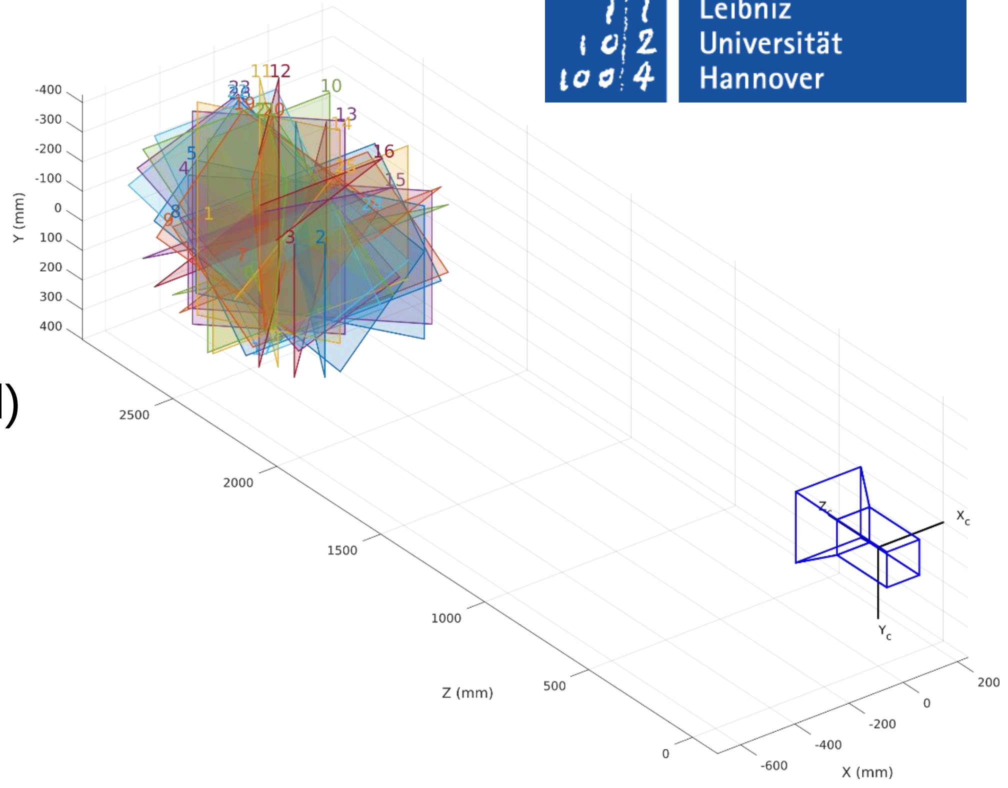
Pose	w($[\phi]$) [deg]	w($[\theta]$) [deg]	w($[\psi]$) [deg]	w($[t_x]$) [cm]	w($[t_y]$) [cm]	w($[t_z]$) [cm]
1	1.4	1.1	1.3	5.3	100.0	3.1
2	0.7	1.2	1.4	6.4	64.4	100.0
3	1.5	2.0	2.3	10.8	7.4	4.7







Results



- Simulated data, 27 checkerboard poses, distance ~2.5 m
- Laser error: [-3,3] cm; [-0.03,0.03] deg (horizontal and vertical)
- Camera error: [-0.3,0.3] px

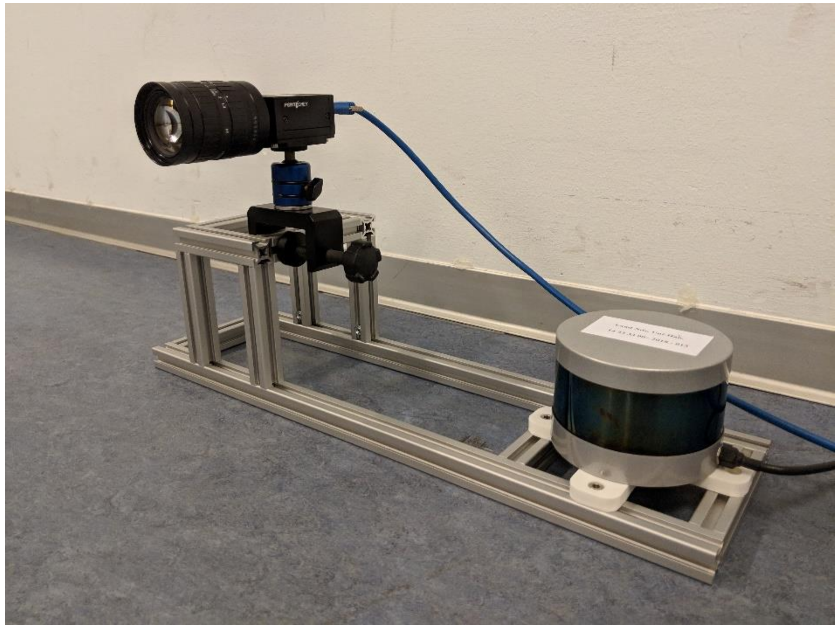
- Guaranteed enclosure for extrinsic parameters
- No initial values needed

	$[\phi]$(Roll) [deg]	$[\theta]$ (Pitch) [deg]	$[\psi]$ (Yaw) [deg]	$[t_x]$ [cm]	$[t_y]$ [cm]	$[t_z]$ [cm]
True	90.0	0.0	0.0	-27.0	15.0	-12.0
Interval	[89.6,90.3]	[-0.4,0.3]	[-0.1,0.3]	[-28.8,-25.0]	[13.1,16.7]	[-13.1,-11.0]

w($[\phi]$) [deg]	w($[\theta]$) [deg]	w($[\psi]$) [deg]	w($[t_x]$) [cm]	w($[t_y]$) [cm]	w($[t_z]$) [cm]
0.7	0.7	0.4	3.8	3.6	2.1

Results

- Real data, 35 checkerboard poses, distance 2 - 3 m
- Laser error: [-3,3] cm; [-0.09,0.09] deg (horizontal); [-0.045,0.045] deg (vertical)
- Camera error: [-0.5,0.5] px



Method	$[\phi]$(Roll) [deg]	$[\theta]$ (Pitch) [deg]	$[\psi]$ (Yaw) [deg]	$[t_x]$ [cm]	$[t_y]$ [cm]	$[t_z]$ [cm]
Interval	[88.3,91.2]	[-1.2,0.1]	[0.1,1.3]	[-27.6,-22.8]	[10.1,21.0]	[-14.2,-8.6]
[Zhou2018]	89.8	-0.1	0.7	-26.8	16.0	-11.6

w($[\phi]$) [deg]	w($[\theta]$) [deg]	w($[\psi]$) [deg]	w($[t_x]$) [cm]	w($[t_y]$) [cm]	w($[t_z]$) [cm]
2.9	1.3	1.2	4.8	10.9	5.6

References

[Zhou2018] L. Zhou, Z. Li, and M. Kaess, “Automatic Extrinsic Calibration of a Camera and a 3D LiDAR Using Line and Plane Correspondences”, in 2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Oct. 2018.

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Results

- How many checkerboard poses are needed for an accurate calibration?
  - Interval analysis allows to determine which poses are redundant or required
- 6 poses vs. 27 poses
  - Comparable accuracy, but less time for the experiment (and less computation time)

Poses	$[\phi]$(Roll) [deg]	$[\theta]$ (Pitch) [deg]	$[\psi]$ (Yaw) [deg]	$[t_x]$ [cm]	$[t_y]$ [cm]	$[t_z]$ [cm]
6	[89.6,90.3]	[-0.4,0.5]	[-0.1,0.3]	[-29.6,-25.0]	[12.7,17.0]	[-13.1,-11.0]
27	[89.6,90.3]	[-0.4,0.3]	[-0.1,0.3]	[-28.8,-25.0]	[13.1,16.7]	[-13.1,-11.0]

Poses	$w([\phi])$ [deg]	$w([\theta])$ [deg]	$w([\psi])$ [deg]	$w([t_x])$ [cm]	$w([t_y])$ [cm]	$w([t_z])$ [cm]
6	0.7	0.9	0.4	4.6	4.3	2.1
27	0.7	0.7	0.4	3.8	3.6	2.1