Institute of Systems Engineering
Real Time Systems Group

Leibniz Universität Hannover

Set-Membership Extrinsic Calibration of a 3D LiDAR and a Camera

Raphael Voges and Bernardo Wagner

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DFG Research Training Group (RTG 2159)
i.c.sens - Integrity and Collaboration in Dynamic Sensor Networks

Motivation: Complementary Sensors

- Camera and laser scanner
  - Complementary information

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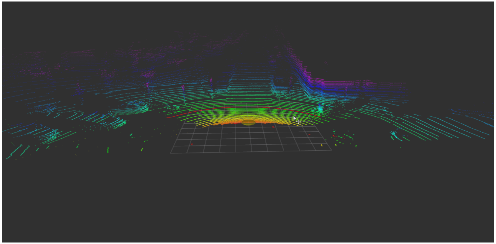

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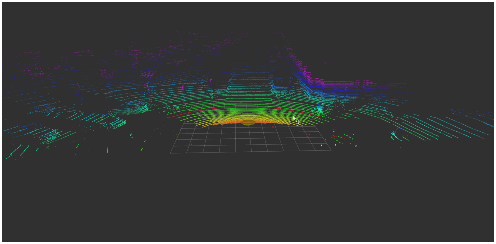
- Extrinsic calibration parameters required to fuse information
  - Rigid body transformation



Motivation: Complementary Sensors

- Camera and laser scanner
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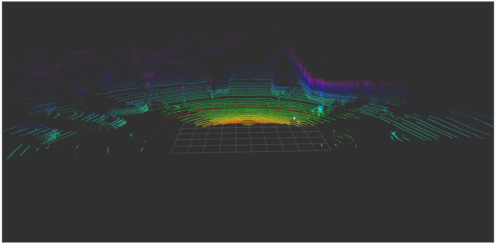
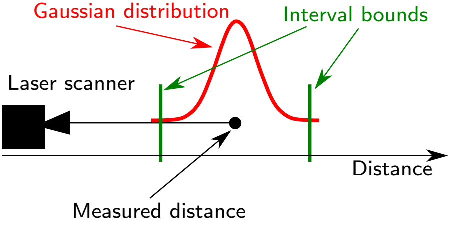
- Extrinsic calibration parameters required to fuse information
  - Rigid body transformation
  - Uncertainties must be taken into account



Motivation: Why Unknown But Bounded Errors?

- Error distribution is often unknown



Motivation: Why Unknown But Bounded Errors?

- Error distribution is often unknown
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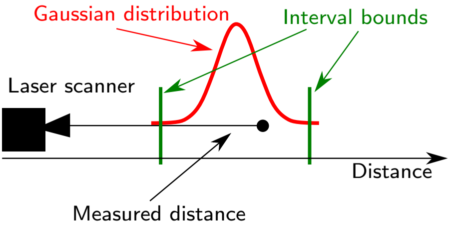

Motivation: Why Unknown But Bounded Errors?

- Error distribution is often unknown
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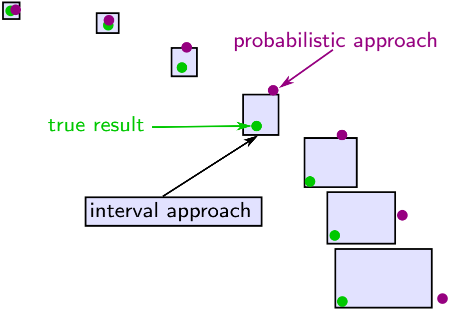
- Results can be guaranteed



Motivation: Why Unknown But Bounded Errors?

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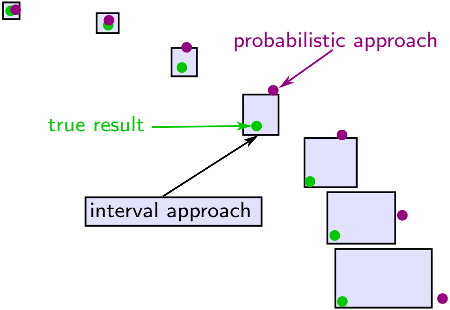
  - Important for safety-critical systems



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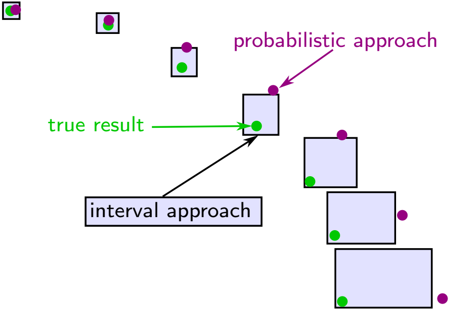

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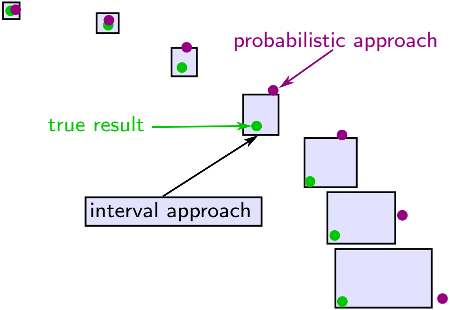

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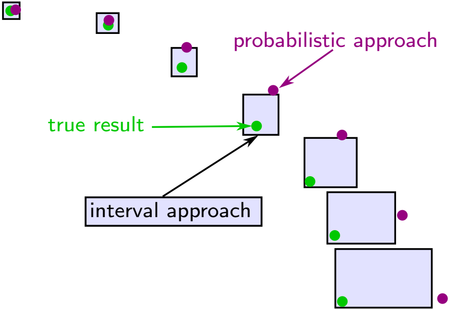
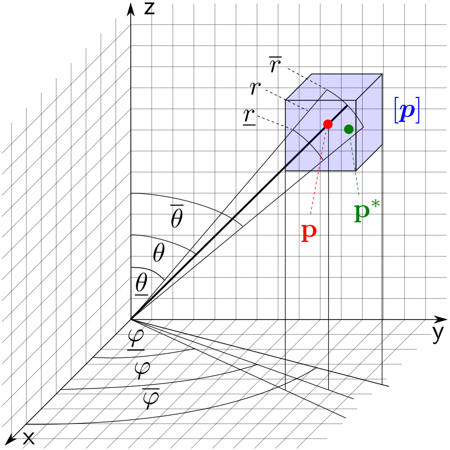
  - Proofs become possible



Bounded Sensor Error Models: Laser Scanner

- Unknown but bounded errors for:



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- Unknown but bounded errors for:
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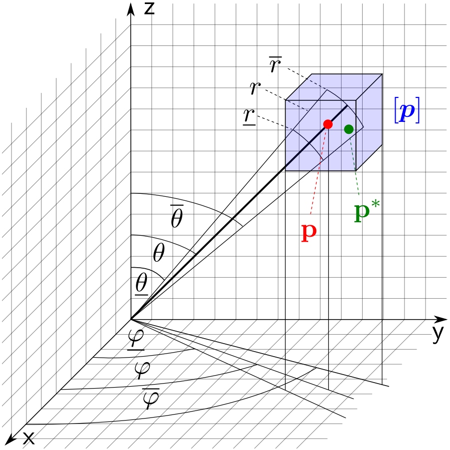

Bounded Sensor Error Models: Laser Scanner

- Unknown but bounded errors for:
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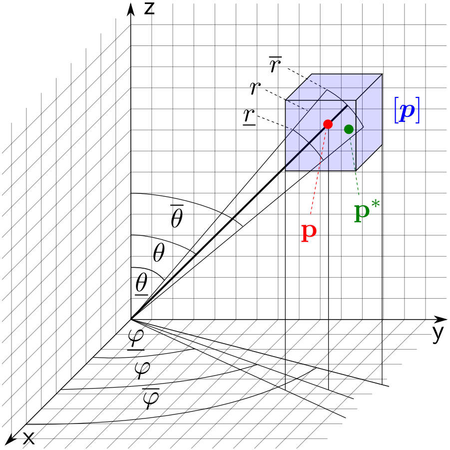
  - Angular components $[\theta] = [\underline{\theta}, \overline{\theta}]$ and $[\varphi] = [\underline{\varphi}, \overline{\varphi}]$



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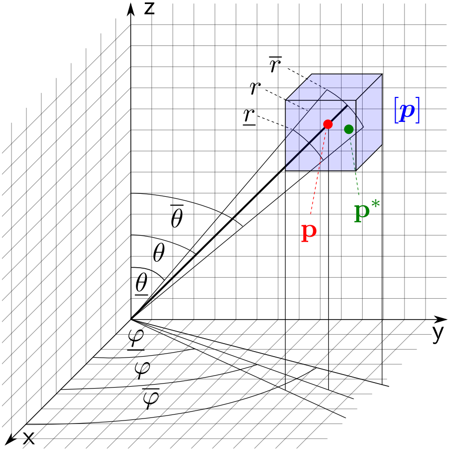

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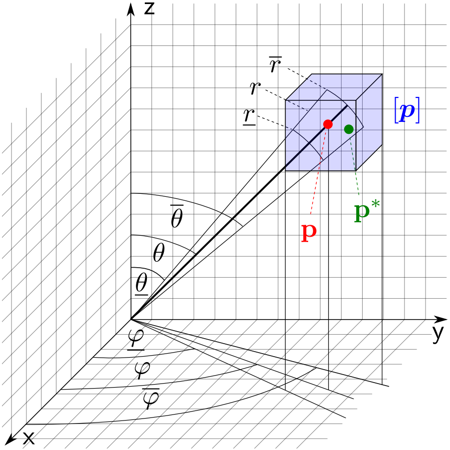
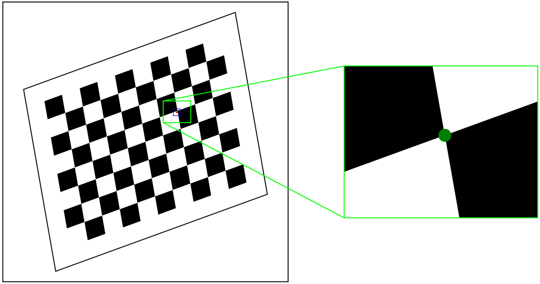
  - Distance measurement $[r] = [\underline{r}, \overline{r}]$
  - Angular components $[\theta] = [\underline{\theta}, \overline{\theta}]$ and $[\varphi] = [\underline{\varphi}, \overline{\varphi}]$
- Conversion into Cartesian coordinates
  - 3D box that is guaranteed to contain the true 3D point: $\mathbf{p}^* \in [\boldsymbol{p}]$



Bounded Sensor Error Models: Camera

- Unknown but bounded errors:
  - Detection of checkerboard corners



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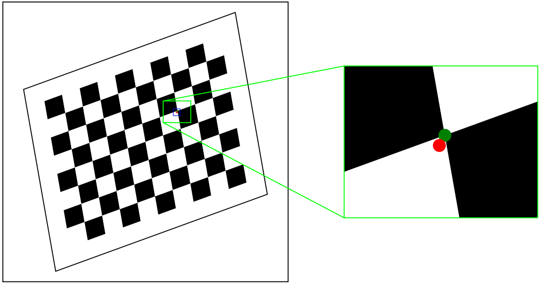

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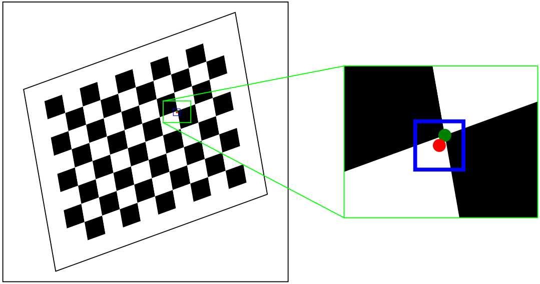
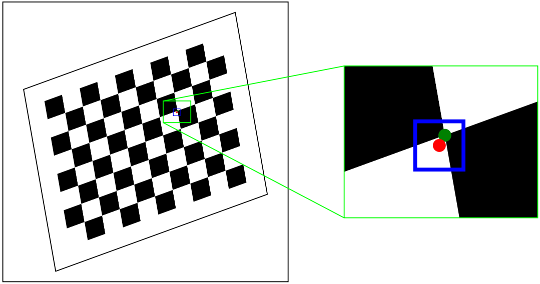


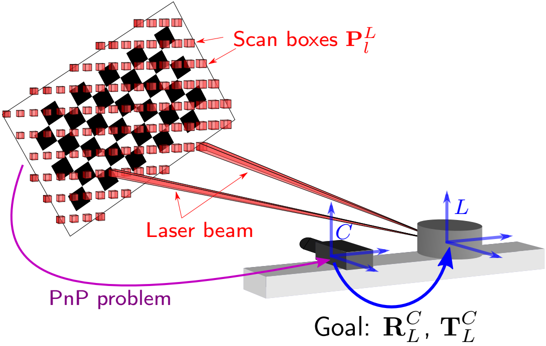
Bounded Sensor Error Models: Camera

- Unknown but bounded errors:
  - Detection of checkerboard corners
- Error bounds:
  - From calibration process (maximum reprojection error)



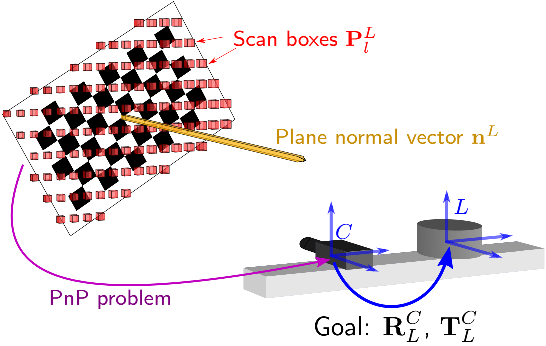
Extrinsic Calibration Between Camera and Laser Scanner

- Solve PnP problem under interval uncertainty [1]
- Extract corresponding features on checkerboard:



Extrinsic Calibration Between Camera and Laser Scanner

- Solve PnP problem under interval uncertainty [1]
- Extract corresponding features on checkerboard:
  - Plane parameters

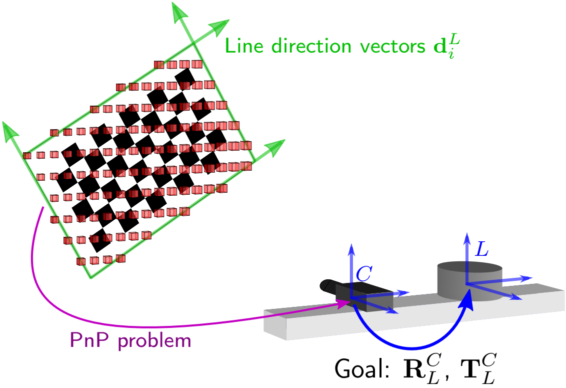


$$1. \quad \mathbf{R}_L^C \mathbf{n}^L = \mathbf{n}^C$$

$$2. \quad \mathbf{n}^C \cdot \left(\mathbf{R}_L^C \mathbf{P}_l^L + \mathbf{T}_L^C \right) + d^C = 0$$

Extrinsic Calibration Between Camera and Laser Scanner

- Solve PnP problem under interval uncertainty [1]
- Extract corresponding features on checkerboard:
  - Plane parameters
  - Boundary line parameters

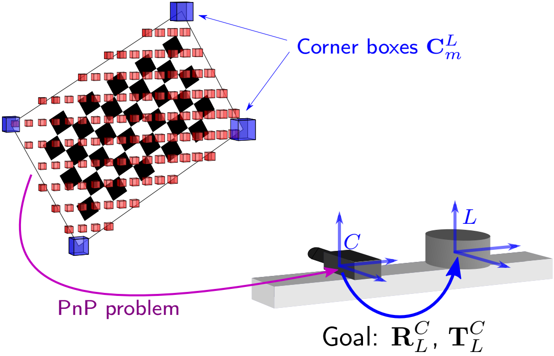


$$3. \quad \mathbf{R}_L^C \mathbf{d}_i^L = \mathbf{d}_i^C$$

$$4. \quad \left(\mathbf{I} - \mathbf{d}_i^C \left(\mathbf{d}_i^C\right)^\intercal\right) \left(\mathbf{R}_L^C \mathbf{Q}_{ij}^L + \mathbf{T}_L^C - \mathbf{Q}_{ik}^C\right) = \mathbf{0}$$

Extrinsic Calibration Between Camera and Laser Scanner

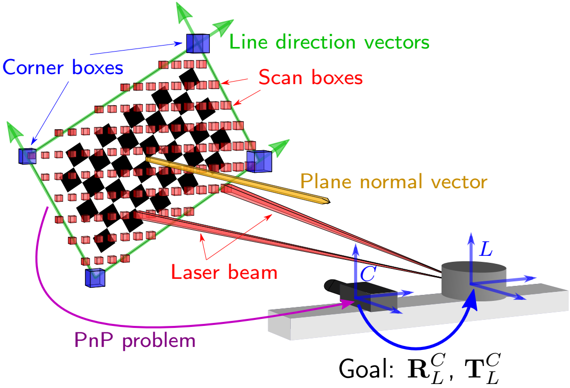
- Solve PnP problem under interval uncertainty [1]
- Extract corresponding features on checkerboard:
  - Plane parameters
  - Boundary line parameters
  - 3D corner points



5. $\mathbf{R}_L^C \mathbf{C}_m^L + \mathbf{T}_L^C = \mathbf{C}_m^C$

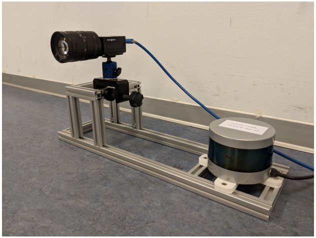
Extrinsic Calibration Between Camera and Laser Scanner

- Solve PnP problem under interval uncertainty [1]
- Extract corresponding features on checkerboard:
  - Plane parameters
  - Boundary line parameters
  - 3D corner points
- Multiple checkerboard poses
- SIVIA with forward-backward contractors to compute $[\mathbf{R}_L^C]$ and $[\mathbf{T}_L^C]$



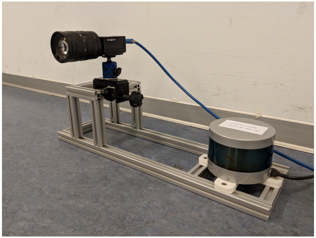
Experimental Results

- Real data



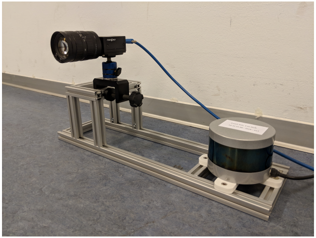
Experimental Results

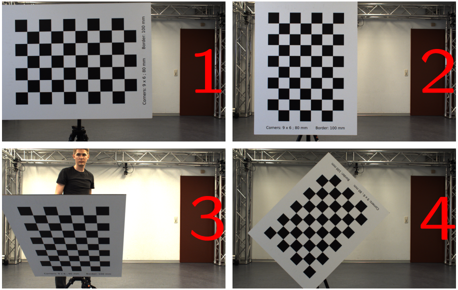
- Real data
- Sensor error bounds
  - Laser scanner: data sheet
  - Camera: intrinsic calibration



Experimental Results

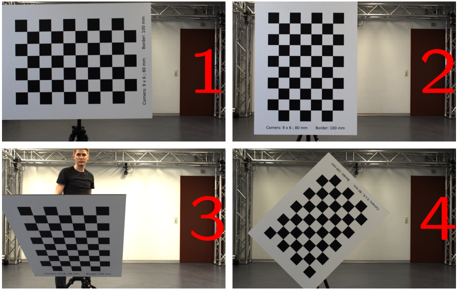
- Real data
- Sensor error bounds
  - Laser scanner: data sheet
  - Camera: intrinsic calibration
- Different checkerboard poses





Experimental Results

- Individual checkerboard poses
- Geometry influences accuracy
- Width (accuracy) of computed parameters



Pose	$w([\phi_L^C])$ (°)	$w([\theta_L^C])$ (°)	$w([\psi_L^C])$ (°)	$w([{}_xT_L^C])$ (cm)	$w([{}_yT_L^C])$ (cm)	$w([{}_zT_L^C])$ (cm)
1	2.9	2.2	1.9	9.8	100.0	**3.5**
2	2.4	2.6	1.6	11.1	100.0	4.2
3	1.0	2.4	2.9	11.1	88.5	**100.0**
4	4.9	5.1	1.0	23.4	21.4	5.9

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Influence of systematic errors

- Simulated data
- Two different error distributions for the laser scanner distance measurement

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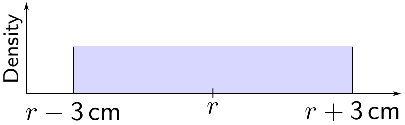

Uniform error distribution with zero-mean error.

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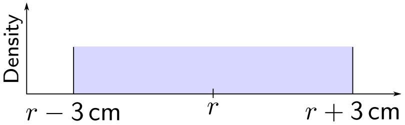

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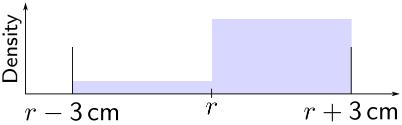

Error distribution exhibiting a systematic error (bias) of 1 cm.

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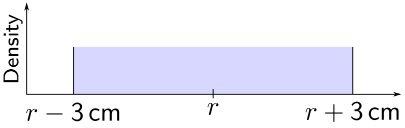

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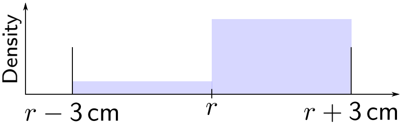

Error distribution exhibiting a systematic error (bias) of 1 cm.

	ϕ_L^C (°)	θ_L^C (°)	ψ_L^C (°)	${}_xT_L^C$ (cm)	${}_yT_L^C$ (cm)	${}_zT_L^C$ (cm)
True	90.0	0.0	0.0	−27.0	15.0	−12.0

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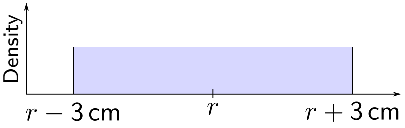

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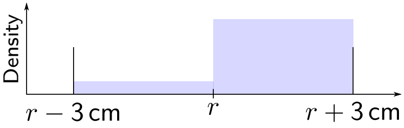

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Zhou [2], no bias	90.01	-0.01	-0.04	-27.00	14.96	-11.91
Our, no bias	$[89.6, 90.3]$	$[-0.4, 0.3]$	$[-0.1, 0.3]$	$[-28.8, -25.0]$	$[13.1, 16.7]$	$[-13.1, -11.0]$

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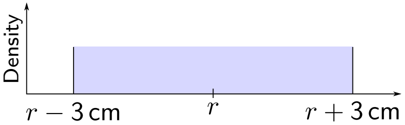

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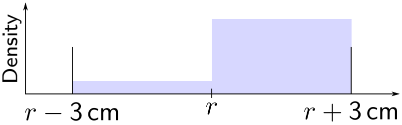

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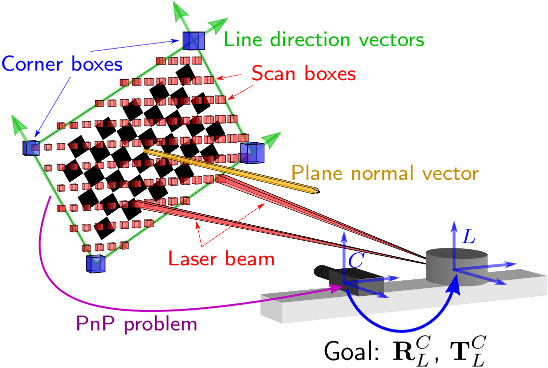

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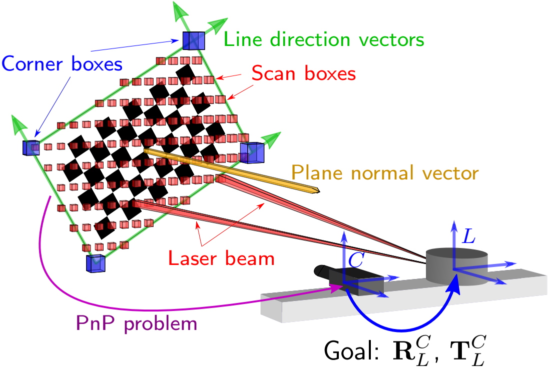
- Calibration parameters sufficiently accurate for sensor fusion



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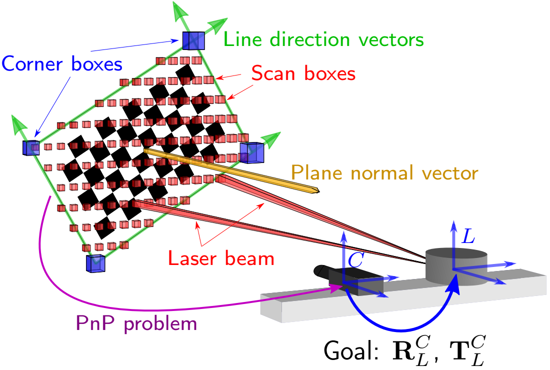
- Calibration parameters sufficiently accurate for sensor fusion
- Approach is compatible with unknown systematic errors



References

[1] R. Voges and B. Wagner, "Timestamp Offset Calibration for an IMU-Camera System Under Interval Uncertainty," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Madrid, Spain, Oct. 2018.

[2] L. Zhou, Z. Li, and M. Kaess, "Automatic Extrinsic Calibration of a Camera and a 3D LiDAR Using Line and Plane Correspondences," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Madrid, Spain, Oct. 2018.

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